SN – 252

V Semester B.A./B.Sc. Examination, November/December 2014
(Semester Scheme) (O.S.) (Prior to 2013-14)
MATHEMATICS – V

ne : 3 Hours

Max. Marks : 90

***Instructions**: Answer **all** the questions.*

I. Answer **any fifteen** of the following : **(15×2=30)**

1) Define a commutative ring. Prove that $(-1)a = -a \ \forall \ a \in R$.

2) Give an example of a division ring which is not a field.

3) Prove that the intersection of two subrings of a ring is a subring.

4) If R is a ring and $a \in R$ then show that $r(a) = \{x \in R / ax = 0\}$ is a right ideal.

5) Prove that a homomorphic image of a commutative ring is commutative.

6) Show that the set $S = \left\{ \begin{pmatrix} a & 0 \\ b & c \end{pmatrix} \Big/ a, b, c \in Z \right\}$ is a subring of the ring $M_2(Z)$ of all 2×2 matrices over the set of integers.

7) Show that $\frac{d}{dt}\left[\vec{f} \times \frac{d\vec{f}}{dt}\right] = \vec{f} \times \frac{d^2\vec{f}}{dt^2}$.

8) Find the unit tangent vector at any point t on the curve $x = 2 \log t$, $y = 4t$, $z = 2t^2 + 1$.

9) Write the serret Frenet formula for the space curve $\vec{r} = \vec{r}(s)$.

10) The cartesian co-ordinates of a point are (2, 2, 1), find the cylindrical co-ordinates.

11) For the curve $x = t$, $y = t^2$ and $z = \frac{2}{3}t^2$, find the equation of the normal plane at $t = 1$.

12) If $\phi(x,y,z) = x^3 - y^3 + xz^2$ find $|\nabla\phi|$ at the point (1, –1, 2).

13) Find the unit normal vector to the surface $x^2 + 2y - 3z^2 = 5$ at (1, 2, 0) on it.

14) If $\vec{F}$ and $\vec{G}$ are irrotational then show that $\vec{F} \times \vec{G}$ is solenoidal.

15) If $\phi = x^2 - y^2 + 4z$. Show that $\nabla^2 \phi = 0$.

16) Find the directional derivative of $\phi(x, y, z) = x^2 - y^2 + 4z^2$ at (1, 1, –8) in the direction of $2\hat{i} + \hat{j} - \hat{k}$.

17) Write the Rodrigue's formula for the Legendre polynomials.

18) Prove that $P_n(-1) = (-1)^n$.

19) Show that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} . \cos x$.

20) Prove that $J_n(x) = \frac{x}{2n}\left[J_{n-1}(x) + J_{n+1}(x)\right]$.

II. Answer **any four** of the following questions : (4x5=

1) Prove that the set S = {0, 1, 2, 3, 4} is a ring under addition modulo 5 and multiplication modulo 5.

2) If R is a ring such that $a^2 = a \; \forall a \in R$.

   Prove that

   i) $a + a = 0 \; \forall \; a \in R$

   ii) $a + b = 0 \Rightarrow a = b$

   iii) R is a commutative ring.

3) Prove that every finite integral domain is a field.

4) Prove that the set of all matrices of the form $\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$ where $a, b \in Z$ is a right ideal of the ring of 2×2 matrices over Z. Show that it is not a left ideal.

5) If $\phi : R \to R'$ is an homomorphism, then prove that

   i) Ker ϕ is a subring of R

   ii) $\phi(R)$ is a subring of R'.

6) Prove that an ideal S of a commutative ring R with unity is maximal if and only if the residue class R/K is a field.

III. Answer **any three** of the following : (3×5=15)

1) For the space curve $x = a \cos t$, $y = a \sin t$, $z = bt$ show that $K = \frac{a}{a^2 + b^2}$ and $\tau = \frac{b}{a^2 + b^2}$.

2) Find the equation of the tangent plane and normal line to the surface $2xz^2 - 3xy - 4x = 7$ at $(1, -1, 2)$.

3) For the space curve $x = t$, $y = t^2$, $z = \frac{2}{3}t^3$, find the unit tangent vector at $t = 1$.

4) Show that the surface $5x^2 - 2yz - 9x = 0$ is orthogonal to the surface $4x^2y + z^3 = 4$ at $(1, -1, 2)$.

5) Show that the cylindrical co-ordinate system is an orthogonal curvilinear co-ordinate system.

IV. Answer **any three** of the following : (3×5=15)

1) If $\vec{F} = 3xyz^2\,\hat{i} + 2xy^3\,\hat{j} - x^2yz\,\hat{k}$ and $\phi(x, y, z) = 3x^2 - yz$ find $\vec{F}.\nabla\phi$.

2) Show that $\vec{F} = (x^2 - yz)\,\hat{i} + (y^2 - zx)\,\hat{j} + (z^2 - xy)\,\hat{k}$ is irrotational. Find ϕ such that $\vec{F} = \nabla\phi$.

3) Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$ where $r = |\vec{r}|$ and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

4) Prove that div (curl $\vec{F}$) = 0 and curl (grad ϕ) = 0.

5) Derive an expression for the gradient of a scalar point function in orthogonal curvilinear co-ordinates.

V. Answer **any two** of the following : (2×5=10)

1) Prove that $x^3 + 2x^2 - x + 1 = \frac{5}{3}P_0(x) - \frac{2}{5}P_1(x) + \frac{4}{3}P_2(x) + \frac{2}{5}P_3(x)$.

2) Show that $\int_{-1}^{+1} P_m(x)P_n(x) = 0$ for $m \neq n$.

3) Show that $\int_{-1}^{+1} x^2 P_{n+1}(x)P_{n-1}(x)dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$.

OR

Prove that $J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{\sin x - x\cos x}{x}\right]$ and

$J_{-\frac{3}{2}}(x) = -\sqrt{\frac{2}{\pi x}}\left[\frac{x\sin x + \cos x}{x}\right]$

4) Show that $\frac{d}{dx}\left[J_n^2 + J_{n+1}^2\right] = \frac{2}{x}\left[nJ_n^2 - (n+1)J_{n+1}^2\right]$.